Physics and Complexity

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Physics

Dictionary definition:

Branch of science concerned with the nature and properties of matter and energy

But today I want to use it as much as

a mind-set with valuable methodologies

And to show application

to many complex systems in many different arenas

Physics

as sometimes portrayed

Particle Physics

'Fundamental' particles

Cosmology

How it all began

Search for the

'Theory of everything'

But not today

'More is different'

~~Particle Physics~~ ~~Cosmology~~

~~'Fundamental' particles~~ ~~How it all began~~

~~'Theory of everything'~~

TOE is by no means the whole story

Many body systems often give new behaviour through co-operation

Both 'fundamental' and applicable

Examples of emergent phenomena

- Superconductivity
- Magnetism
- Giant Magnetoresistence
- Quantum Hall Effect

Useful
& often give very high accuracy

- Superconductivity
  - Flux quantization
- Magnetism
- Giant Magnetoresistence
  - Basis of modern high capacity data storage
- Quantum Hall Effect
  - Quantized conductivity plateaux

Highest accuracy measurements of fundamental constants
even in dirty systems

Complexity/Complex Systems

- Many body systems
- Cooperative behaviour complex
  - non-trivial and new
  - not simply anticipated from microscopics
  - even with simple individual units
  - and simple interaction rules
- But with surprising conceptual similarities between superficially different systems

Typical approach

- Essentials?
  - Minimal models
  - Comparisons/checks: e.g. simulation
  - Analysis: maths & ansätze
- Important consequences?
- Universalities?

Build → • Conceptualization

→ • Generalization

• Application

Key ingredients

Frustration

Conflicts

Disorder

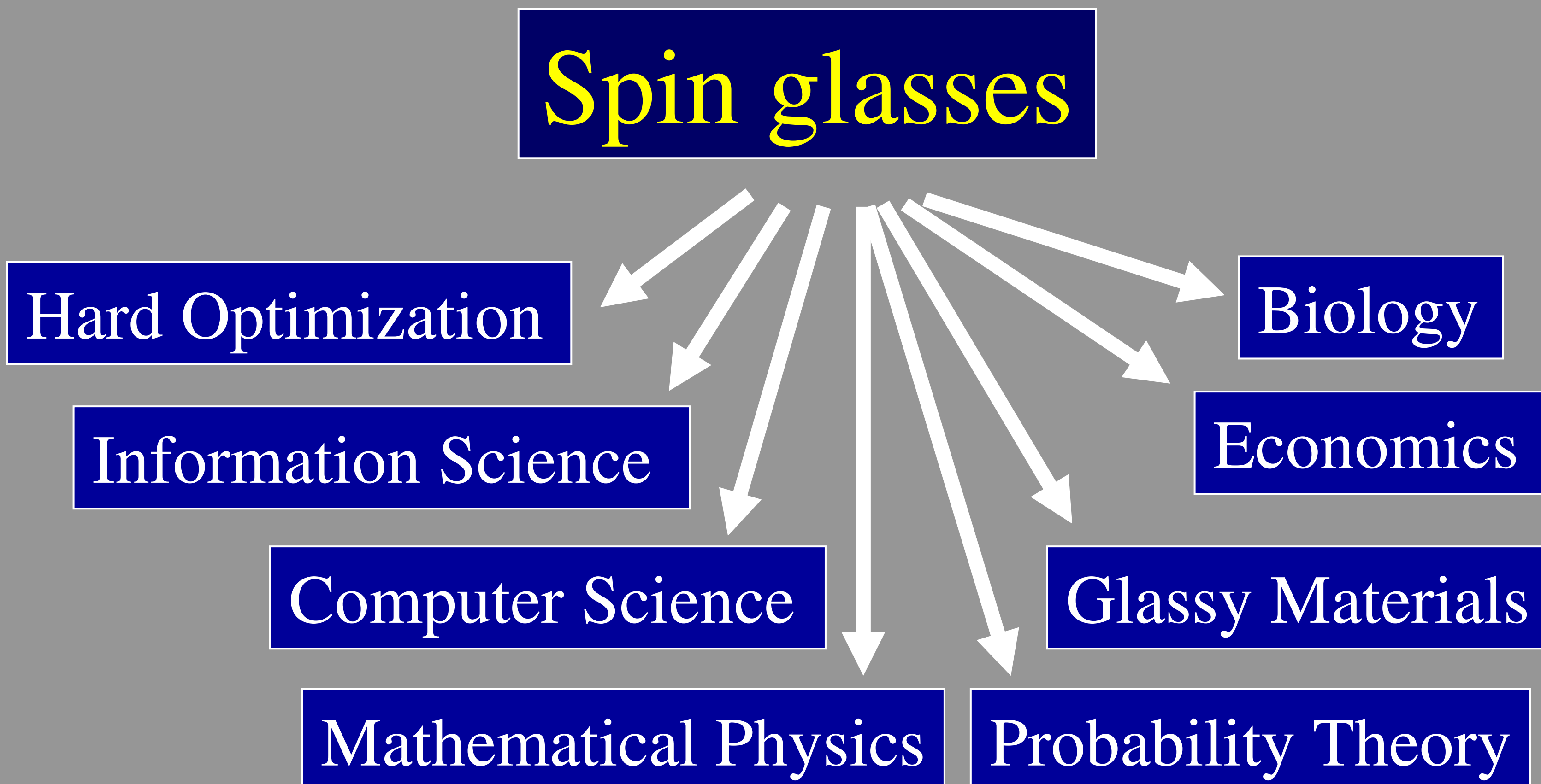
Frozen

or time-dependent; e.g. uncertainty

Emphasis

- Novel physics
- New concepts
- Minimalist models
- Interdisciplinary transfers
- Much ubiquity, some differences
- Relevance of noise and memory
- Applicability

Examples


Spin glasses
Hard Optimization
Information Science
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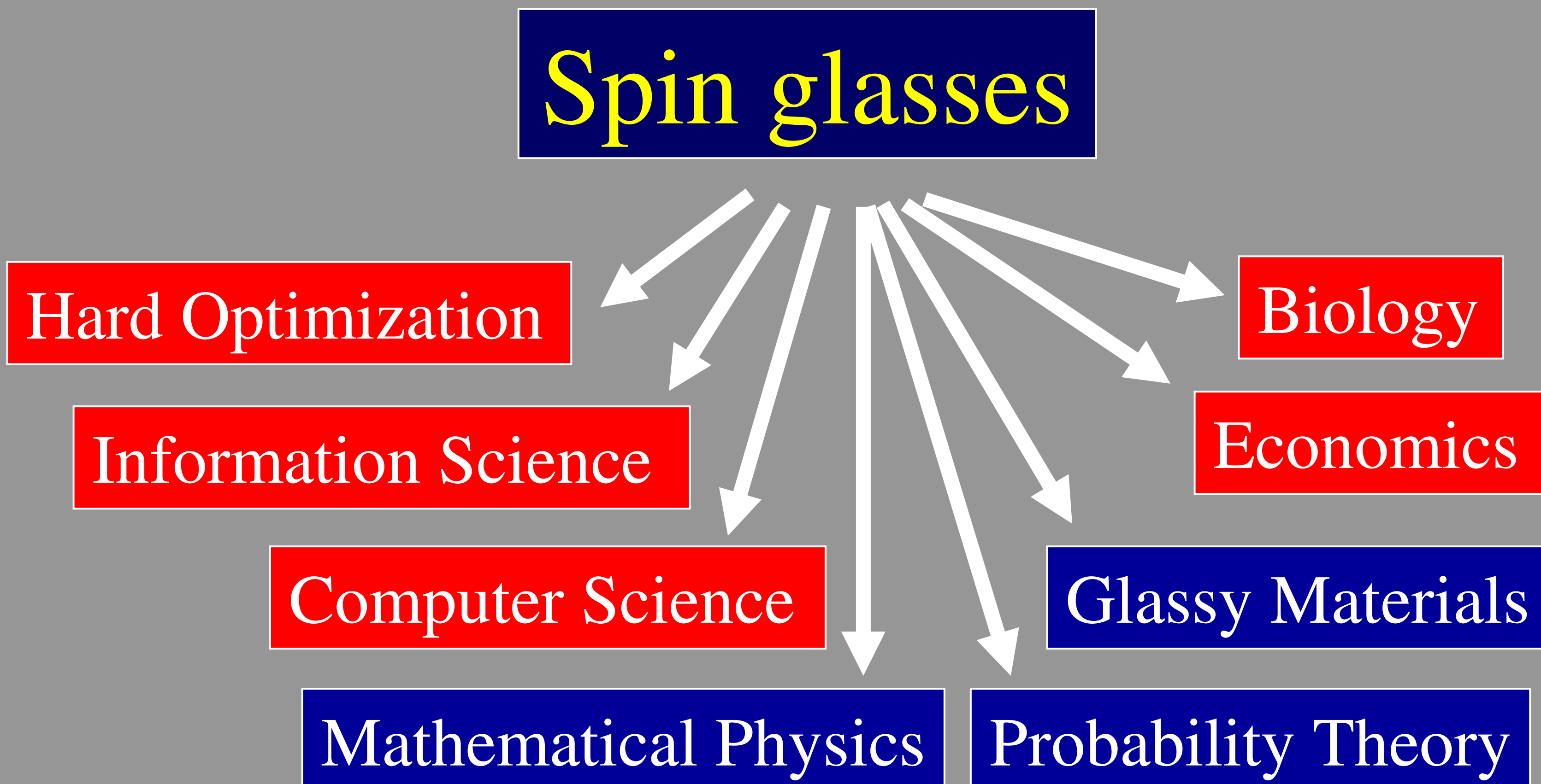
Examples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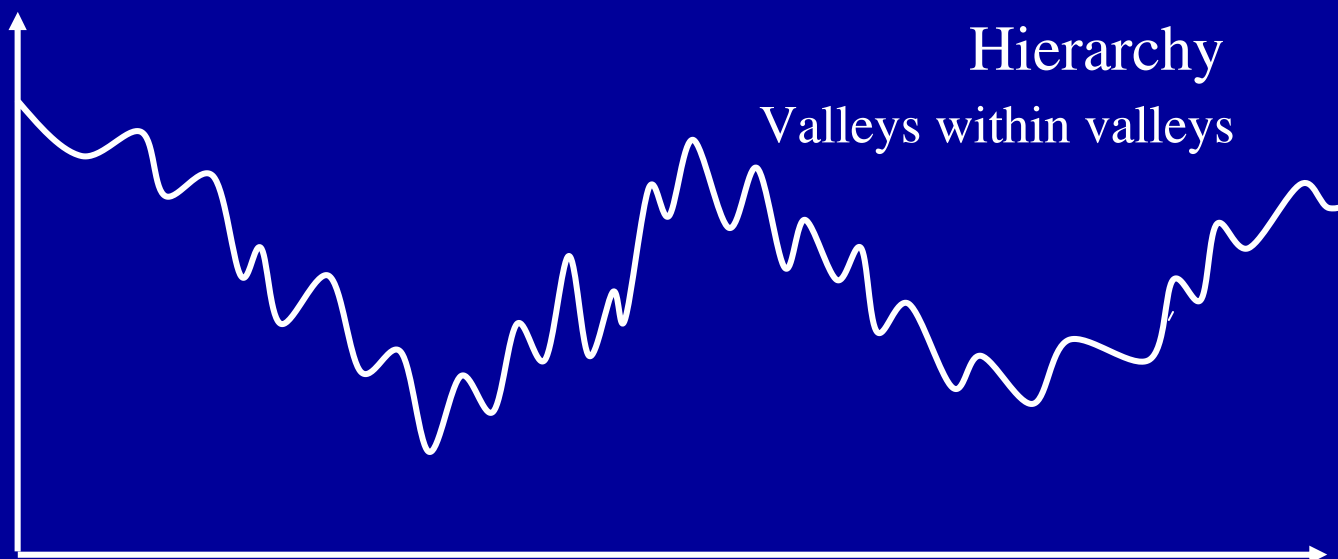
Rugged Landscape Paradigm

Two-dimensional cartoon of high dimensional concept

Many metastable states

Cost

to minimise



Coordinate

Dynamics
c.f. motion

Hard to minimise: sticks: glassiness

General theoretical structure

Control functions

$$F(\{J_{ij....k}\},\{S_{ij..}\},\{T\})$$

Statics: Fixed Variable

Dynamics: Slow Fast

External influences

Control functions, but who controls?

- Physics: nature/physical laws
- Biology: nature but not necess. equilibrium
- Hard optimization: we choose algorithms
- Information science: we have choice
- Markets: partly supervising bodies, partly manufacturers, partly speculators
- Society: governments can change rules

Physics: Magnets: Spin glasses

- Disordered magnetic alloys *e.g.* $Au_{1-x}Fe_x$
  - Competitive magnetic interactions

  - No periodicity → no simple best compromise

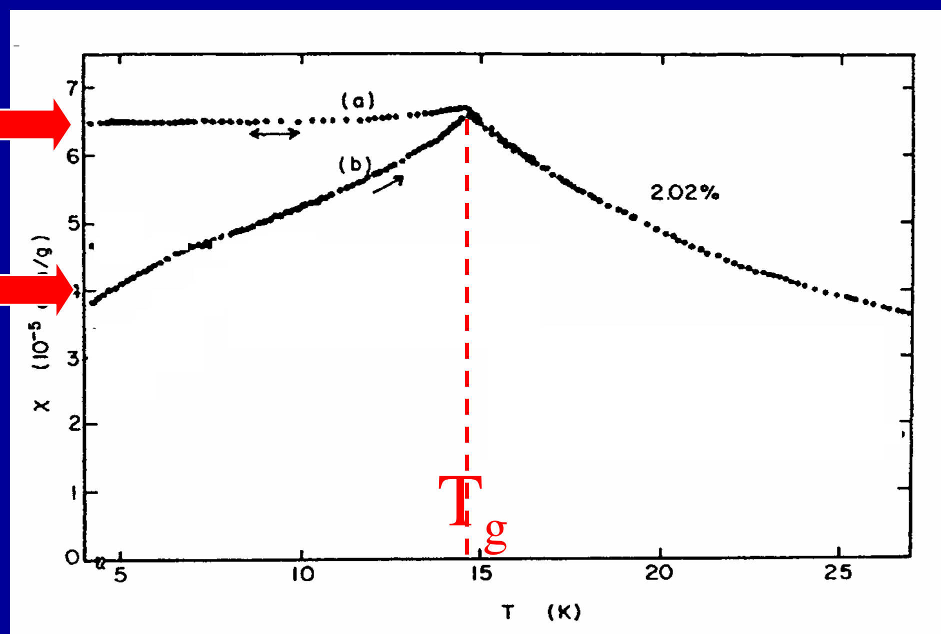
- Non-periodic magnetic moment freezing
- Slow macrodynamics/ history-dependence/ aging
- Similar for site or bond disorder

Phase transitions & preparation-dependence

Susceptibility

Field-cooled

Zero-field cooled



AuFe

non-equilibrium equilibrium

Minimalist Model

Cost or Hamiltonian

$$H = -\sum_{(ij)} J_{ij} s_i s_j$$

$s = \pm 1$

Spin up/down

Magnetic elements

Quenched random interaction: ±

Frustration & Disorder

Minimalist Model

$$H = -\sum_{(ij)} J_{ij} s_i s_j$$

Simulations

~ experiment

Range-free case

soluble but very subtle

"The Dean's Problem"

Allocate N students to 2 residences with maximum happiness

Satisfaction

To maximise

$$H = +\sum_{(ij)} J_{ij} s_i s_j$$

$s = \pm 1$

Dorm A/B

Inter-student friendship: ±

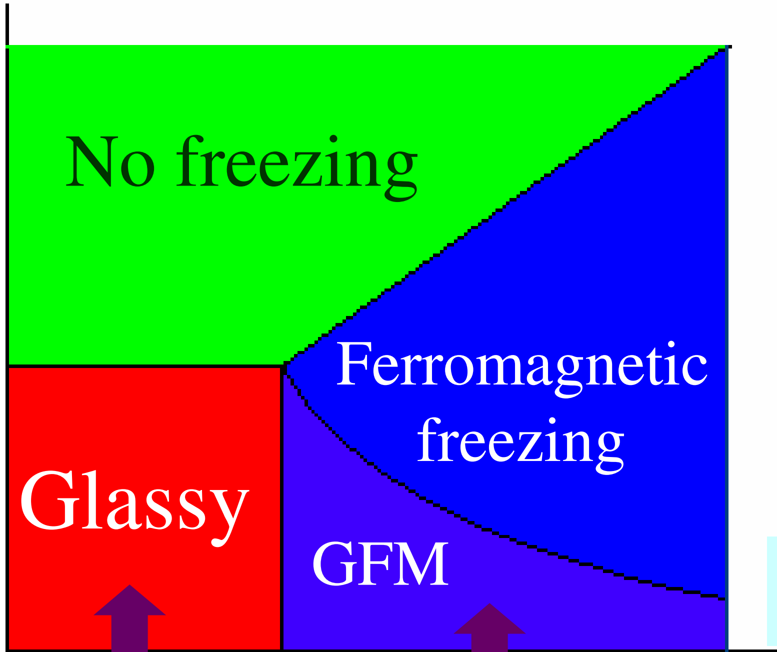
Also $\sum_i s_i = 0$

Phase diagram

Temperature/noise/uncertainty/Dean's impatience

Easy to equilibrate

Hard to equilibrate

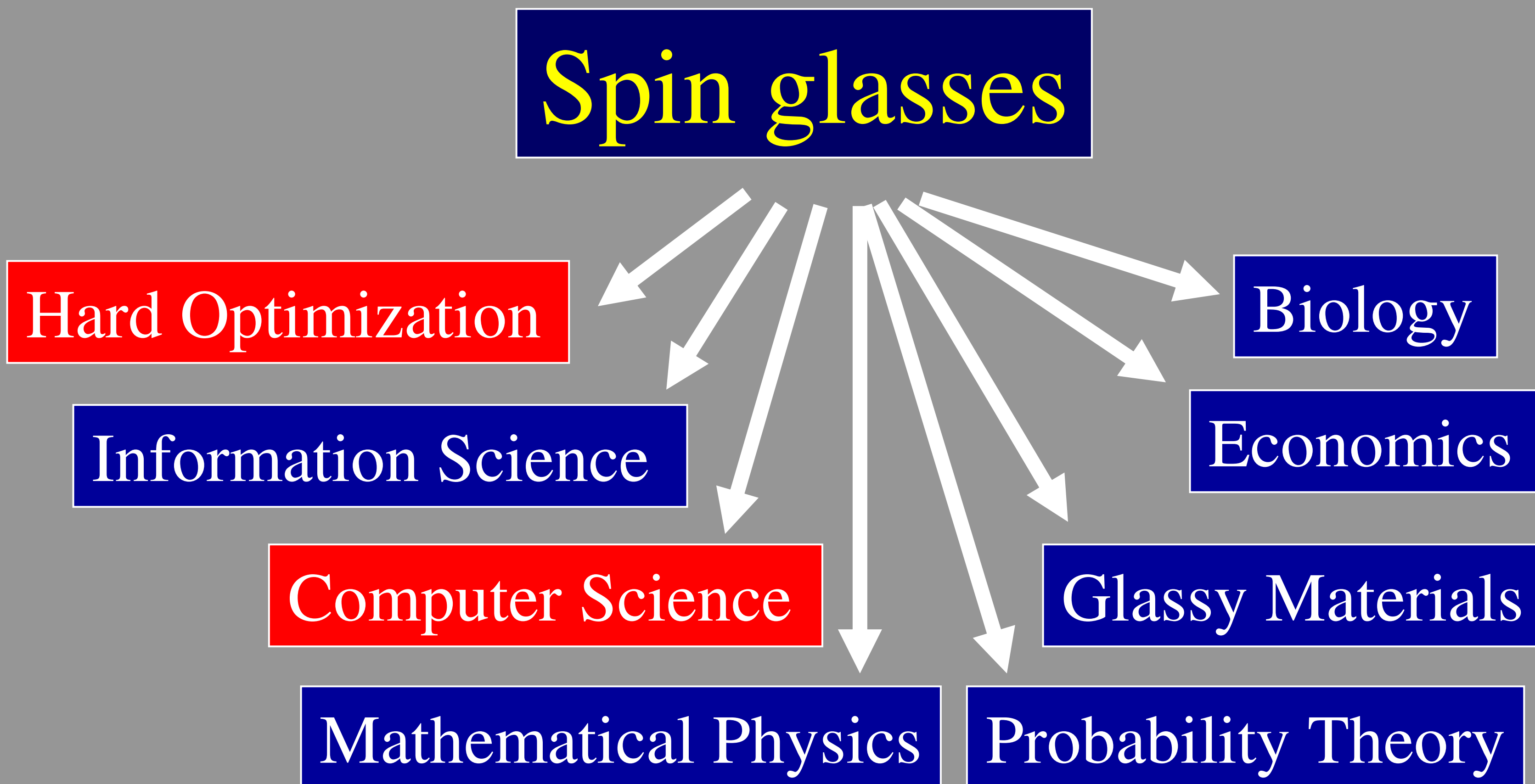


Attractive bias

Many metastable states

'Rugged landscape Slow dynamics

Examples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Examples

- Minimizing a cost
  - *e.g.* distribution of tasks, partitioning
- Satisfiability
  - Simultaneous satisfaction of ‘clauses’
- Error correcting codes
  - Capacity and accuracy

Two issues

- What is achievable?
  - Analogue: “statics”/equilibrium
    - May be hard to find?
    - Is it possible?
- If achievable, how to achieve it?
  - Needs algorithms = dynamics
    - We may be able to devise
    - But glassiness can badly hinder efficacy

Recent example of hard optimization from computer science

K-satisfiability

simultaneous satisfiability
of many 'clauses' of length K

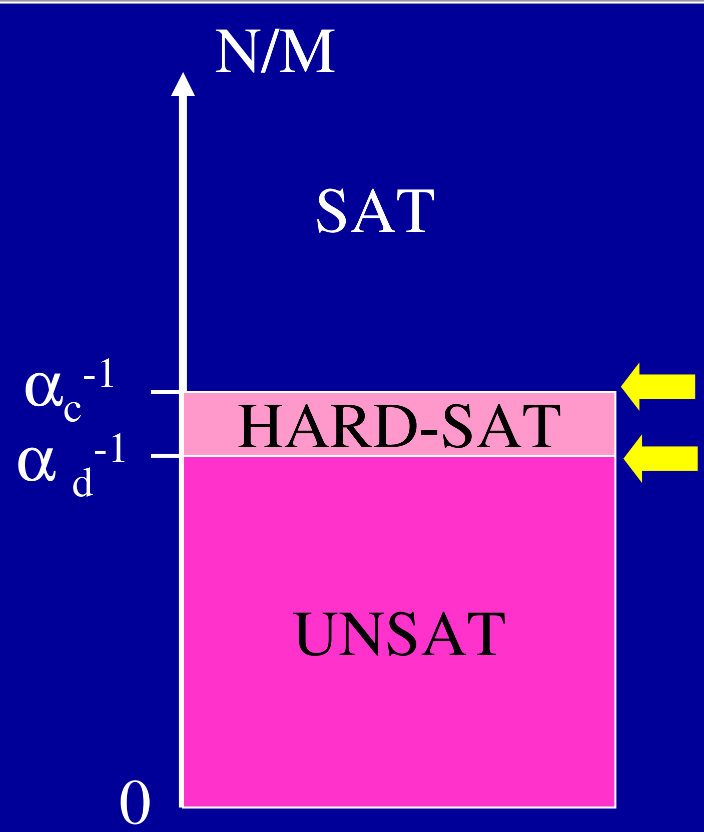
$$(x_1 \text{ or } x_2 \text{ or } \overline{x_3}) \text{ and } (x_3 \text{ or } \overline{x_4} \text{ or } x_5) \text{ and } \ldots$$

$$\alpha \equiv \frac{M}{N} = \left\{ \frac{\text{\# of clauses}}{\text{\# of variables}} \right\}$$

Phase transition(α): SAT / UNSAT

Compare: K-satisfiability

Phase transitions



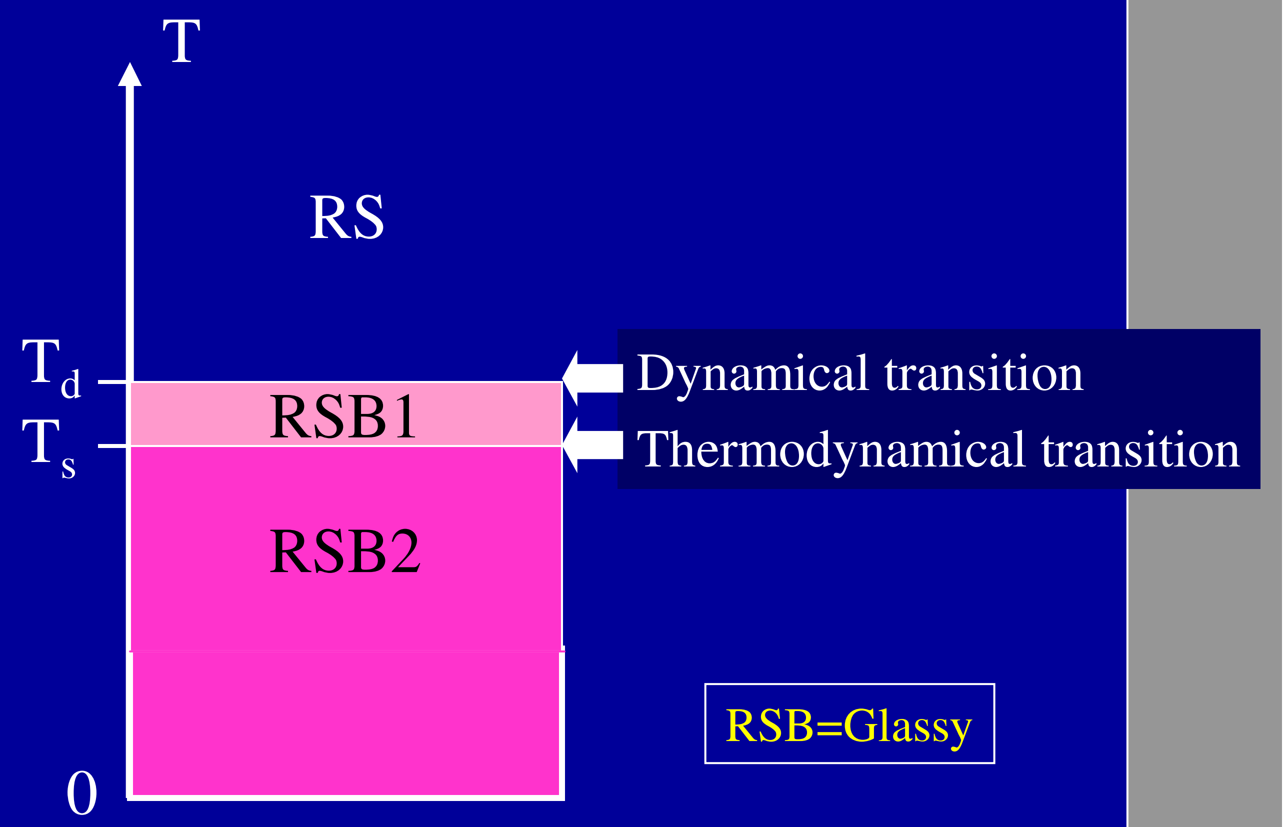
Simple algorithms stick

Theoretically achievable limit

Physicists recognised this subtlety through comparison with *K-spin glass*

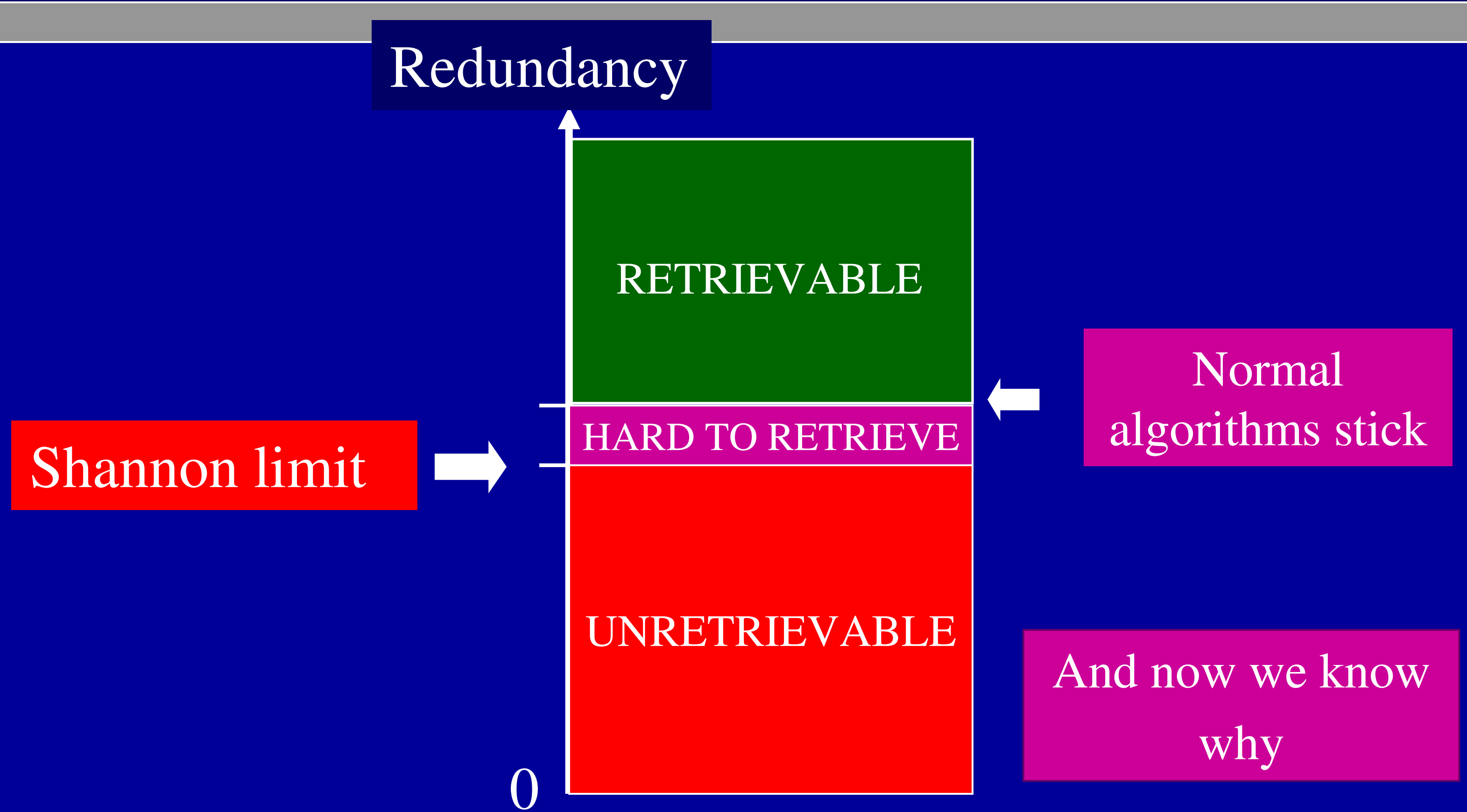
Where the idea came from

Potts or K (>2) -spin glass



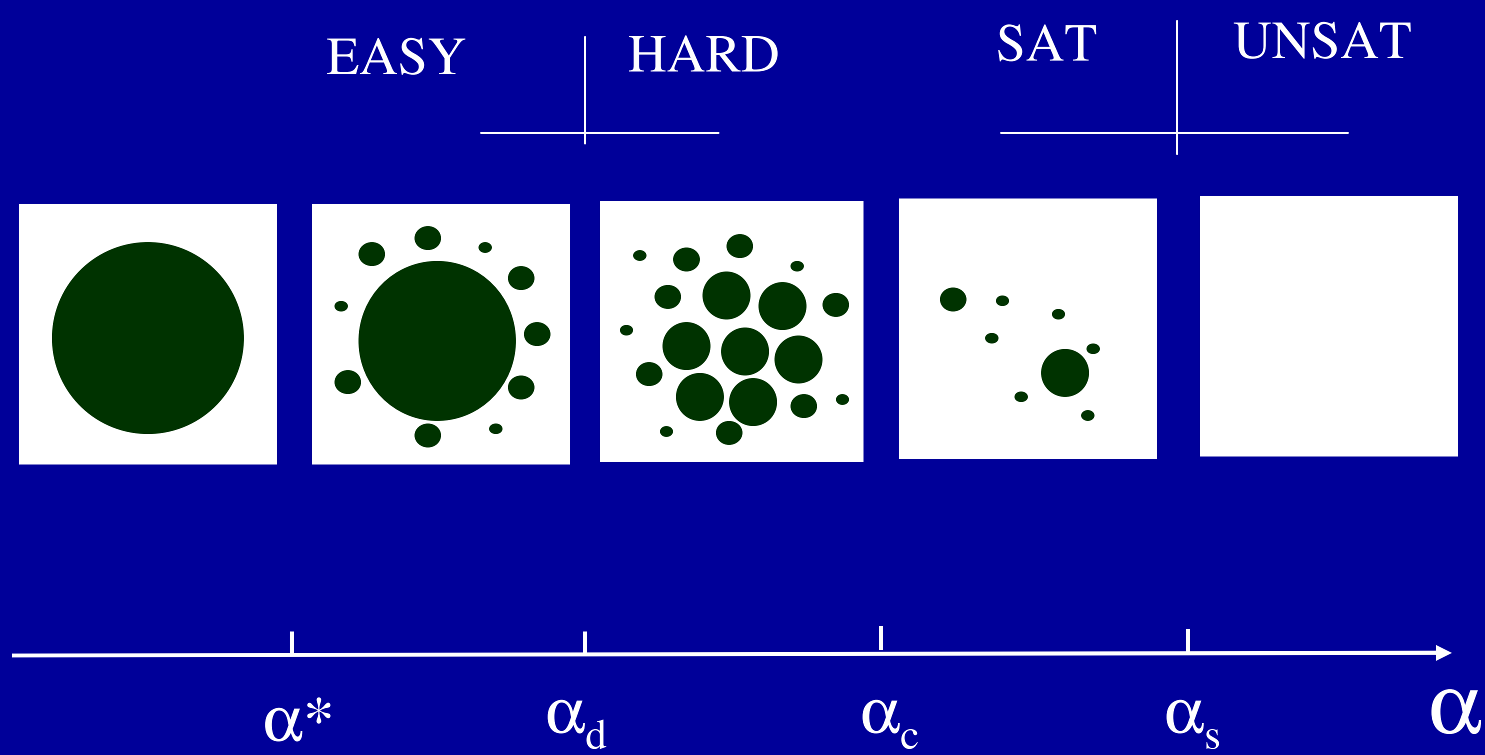
Originally looked at as a purely intellectually interesting extension

Similarly: error-correcting codes



In fact, more regimes

Clustering: Random K-SAT



New algorithms

- Understanding brings opportunities
- Normal physics
  - Algorithms given
- Artificial systems
  - We can design algorithms
    - *e.g.* Computational
      - Simulated annealling
      - Simulated tempering
      - Clustering…….

Great advance: Survey propagation

Simulated annealing

effective stat. mech./thermodynamics

$$Z = \sum_{configurations} \exp(-Cost / kT_{anneal})$$

Artificial 'temperature' T_{anneal}

$$\overline{Min\ Cost} = \lim_{T_A \to 0} T_A \overline{\ln Z}$$

⇓

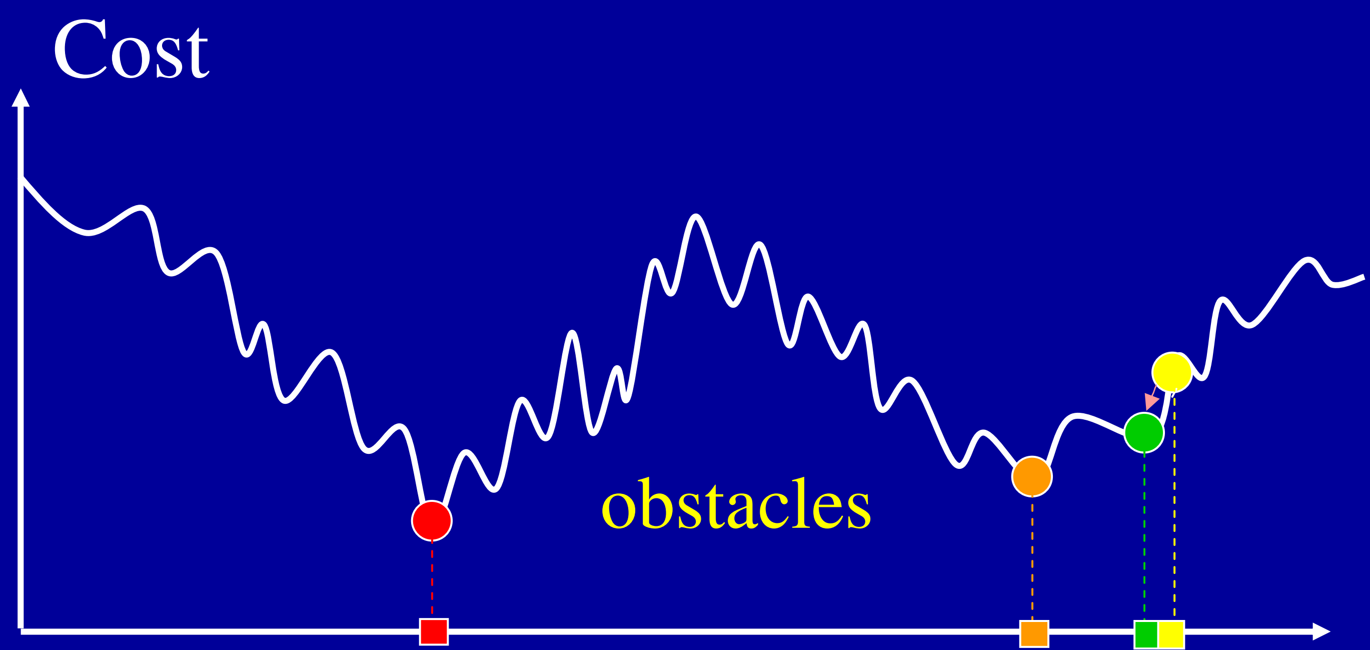
Optimum achievable

Achieving it requires (algorithmic) dynamics

Frustration & disorder → glassiness

But we can choose the dynamics

Landscape paradigm for hard optimization



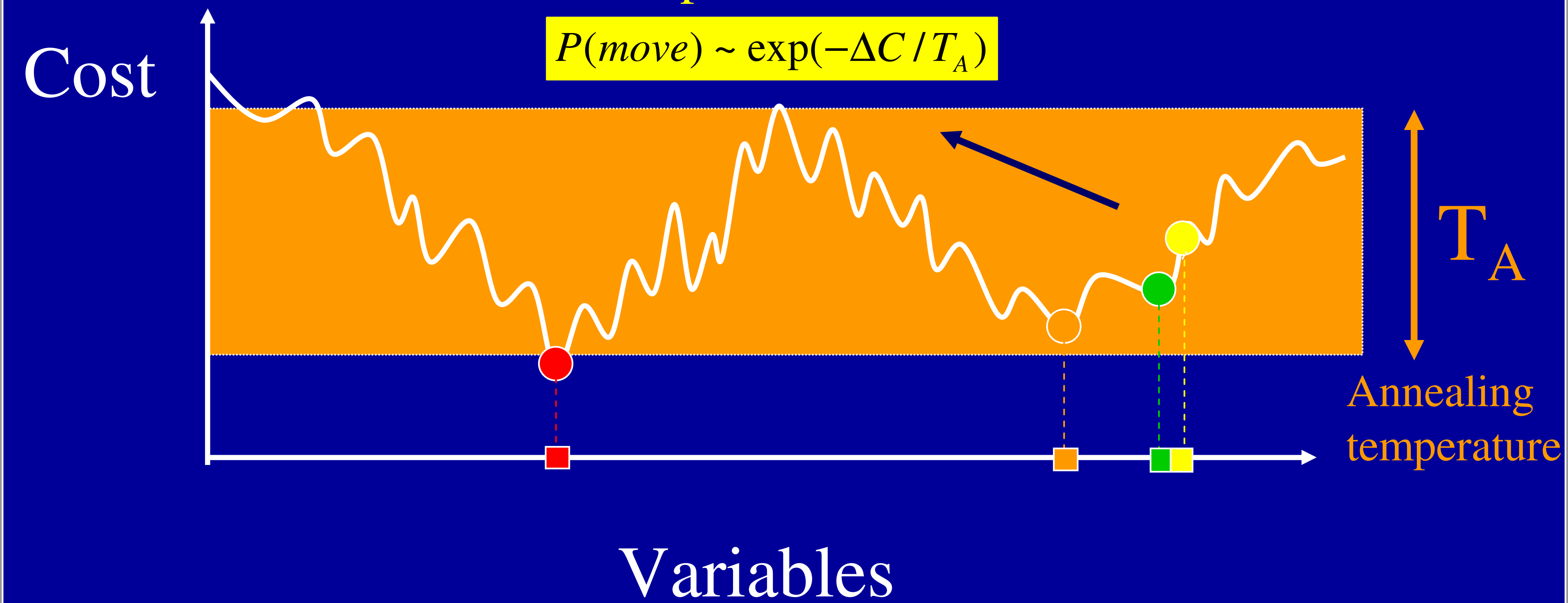
Steepest descent gets stuck

Simulated annealing

Probabilistic hill-climbing

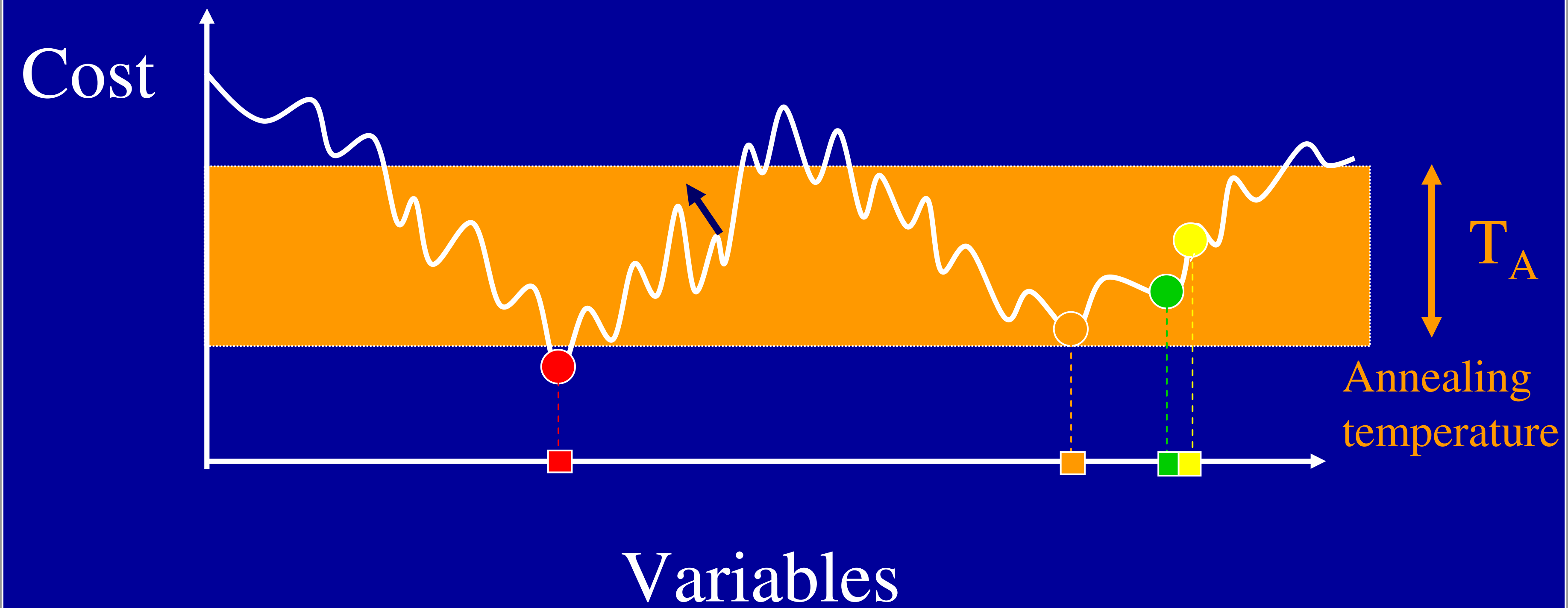
Add 'temperature': freedom

$$P(move) \sim \exp(-\Delta C / T_A)$$



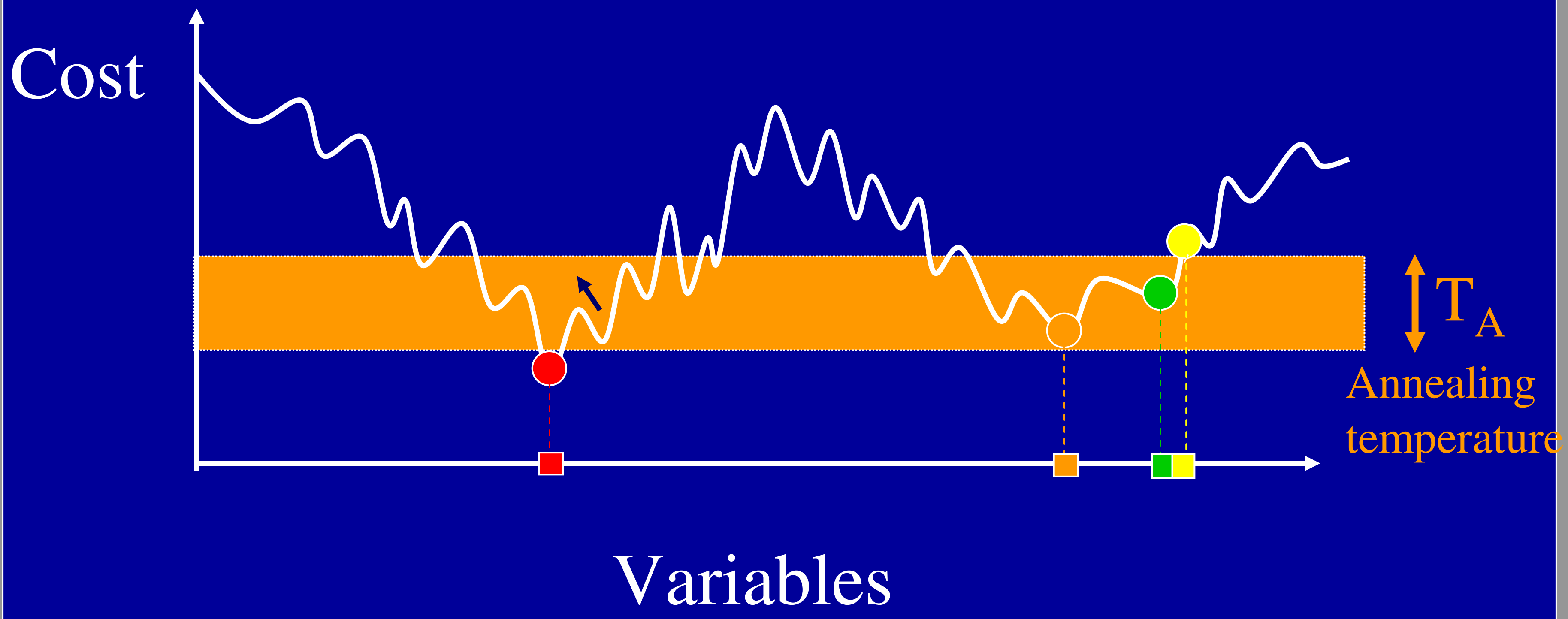
Simulated annealing

Gradually reduce T_A



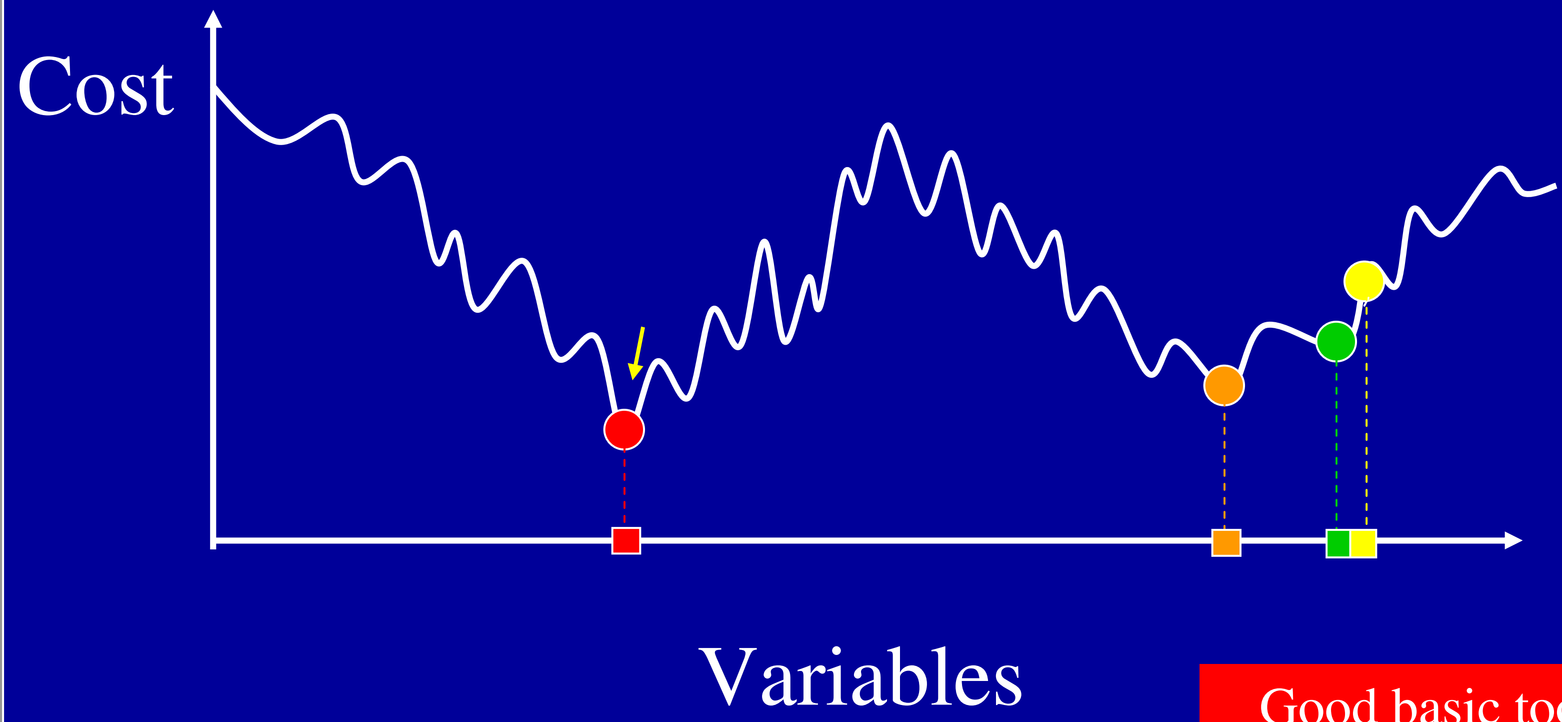
Simulated annealing

Gradually reduce T_A



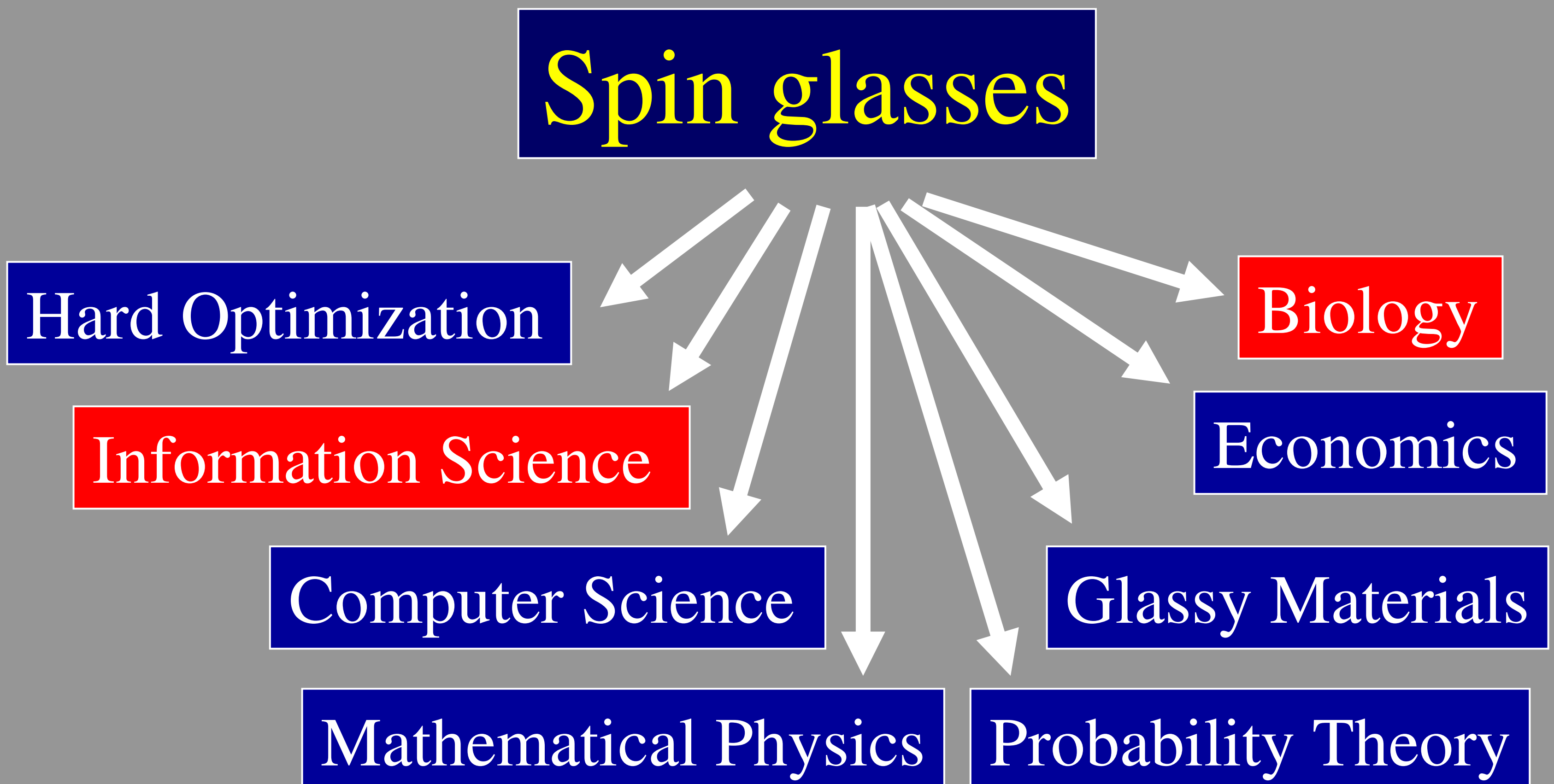
Simulated annealing

Hopefully



Good basic tool
but now better ones

Examples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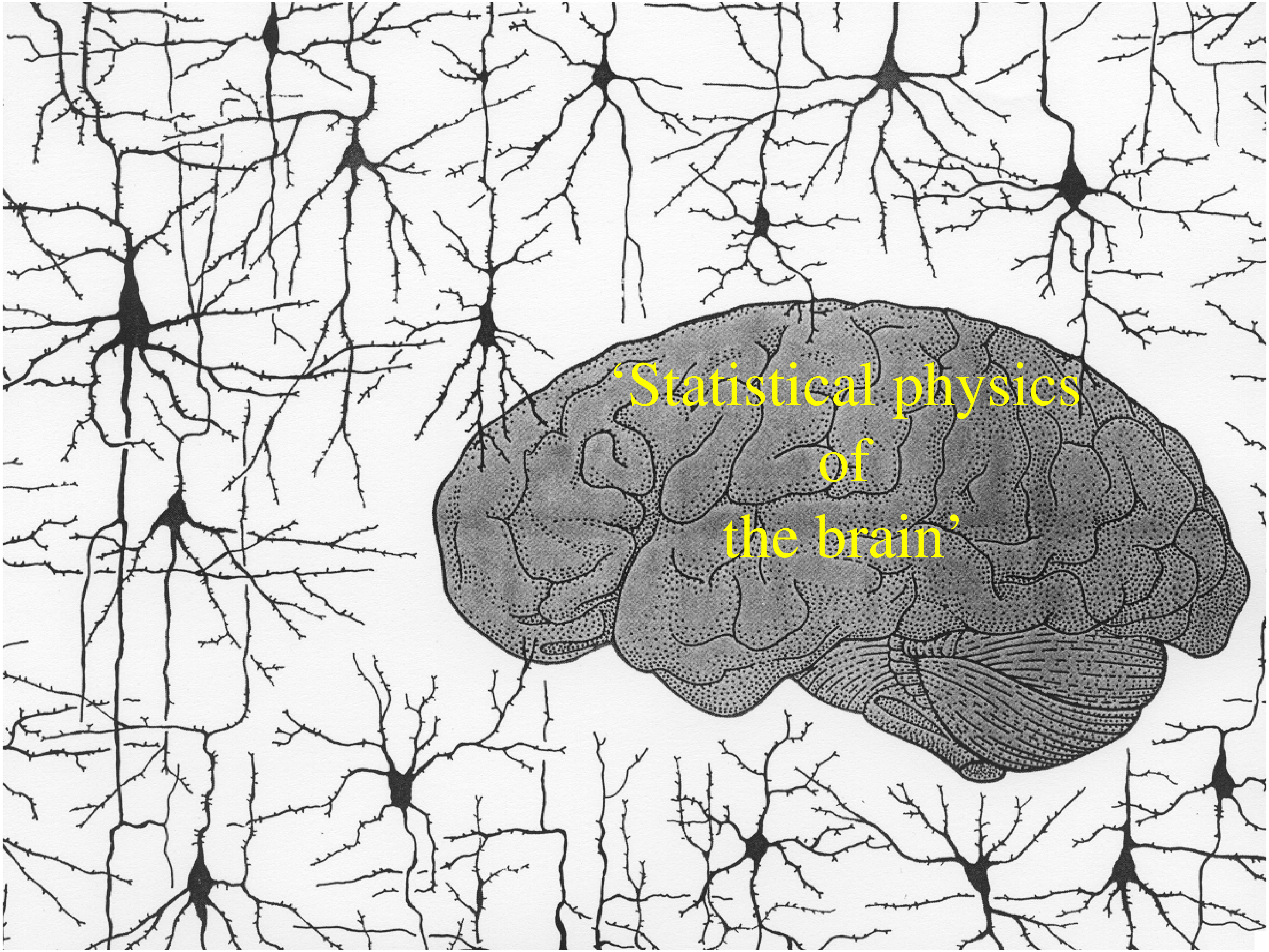
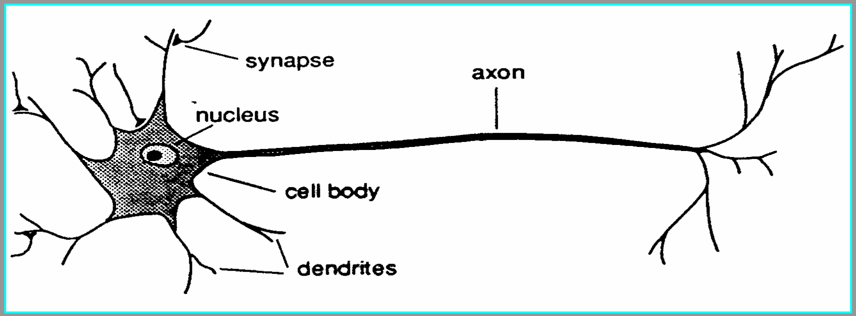
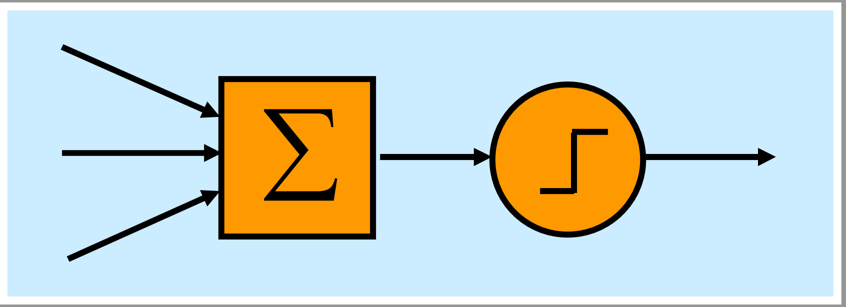

‘Statistical physics
of
the brain’

Typical neuron

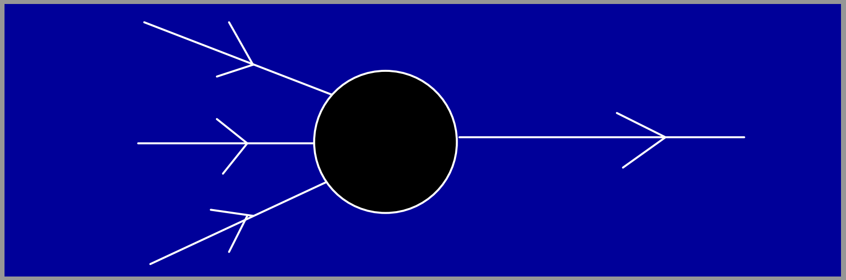


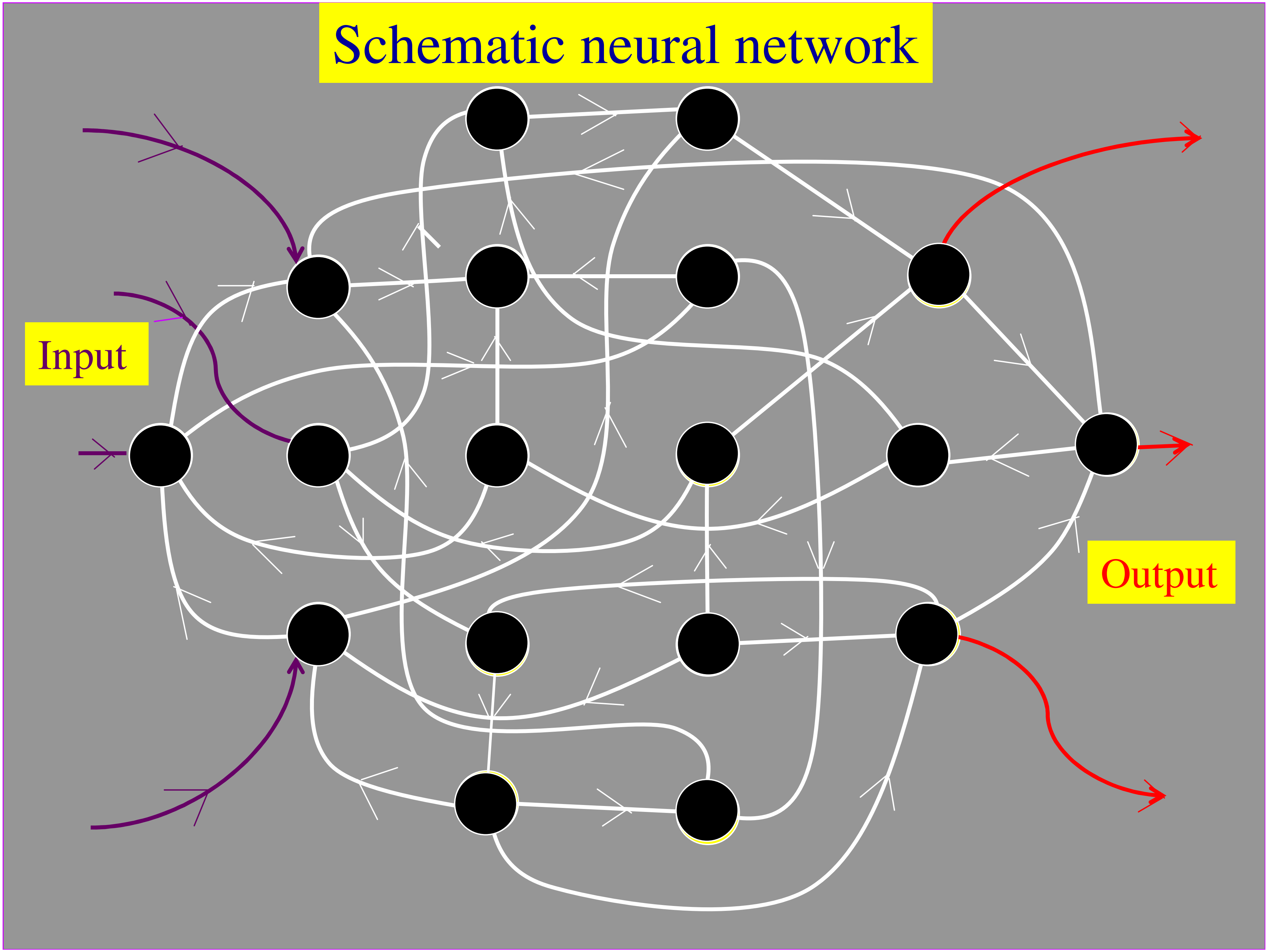
Schematize

(a)

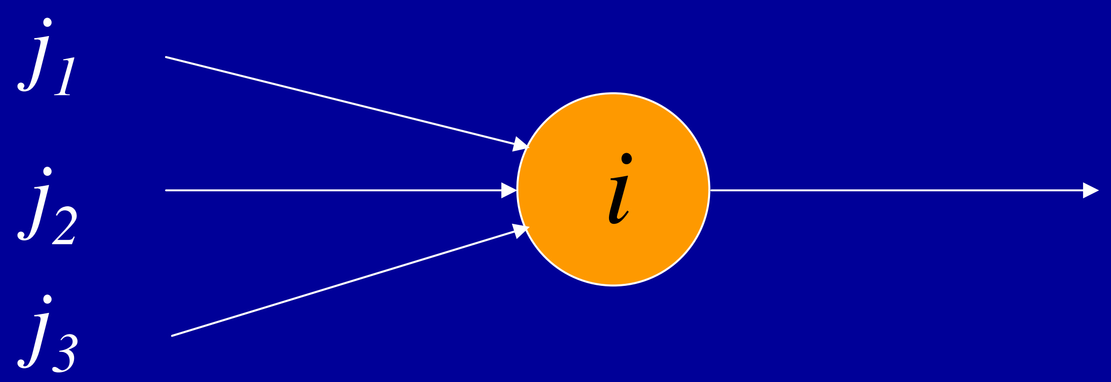


(b)




Schematic neural network
Input
Output

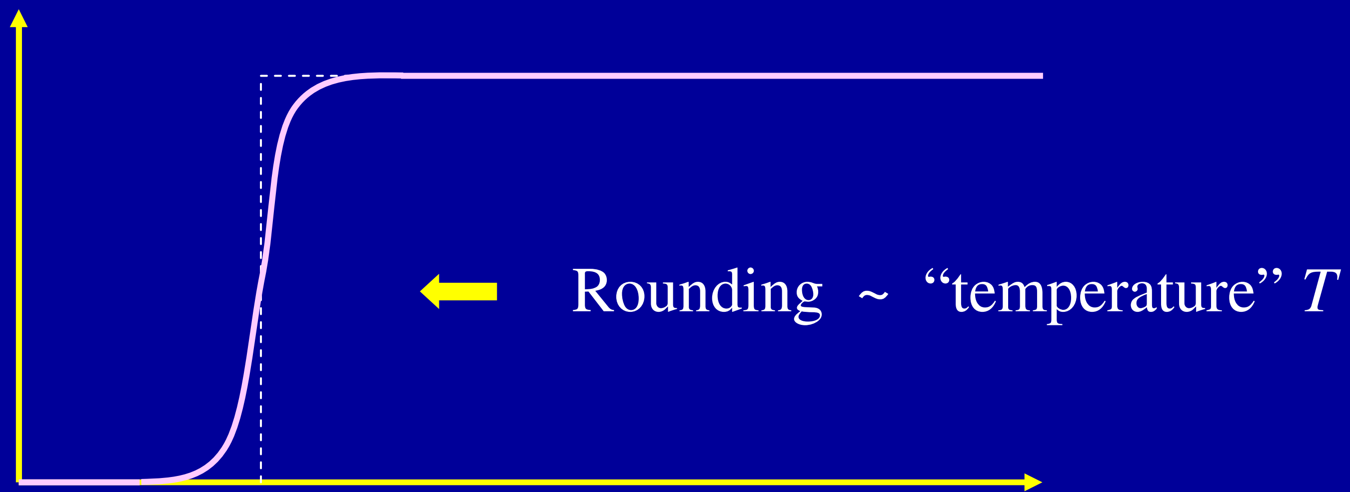
Mathematical modelling



- Neuronal activity: V_i
- Synaptic weights: J_{ij} > 0 switch-on, < 0 switch-off
- Total input: $U_i = \sum_j J_{ij} V_j$

Consequence of input 'potential'

Output activity of neuron/ probability of firing

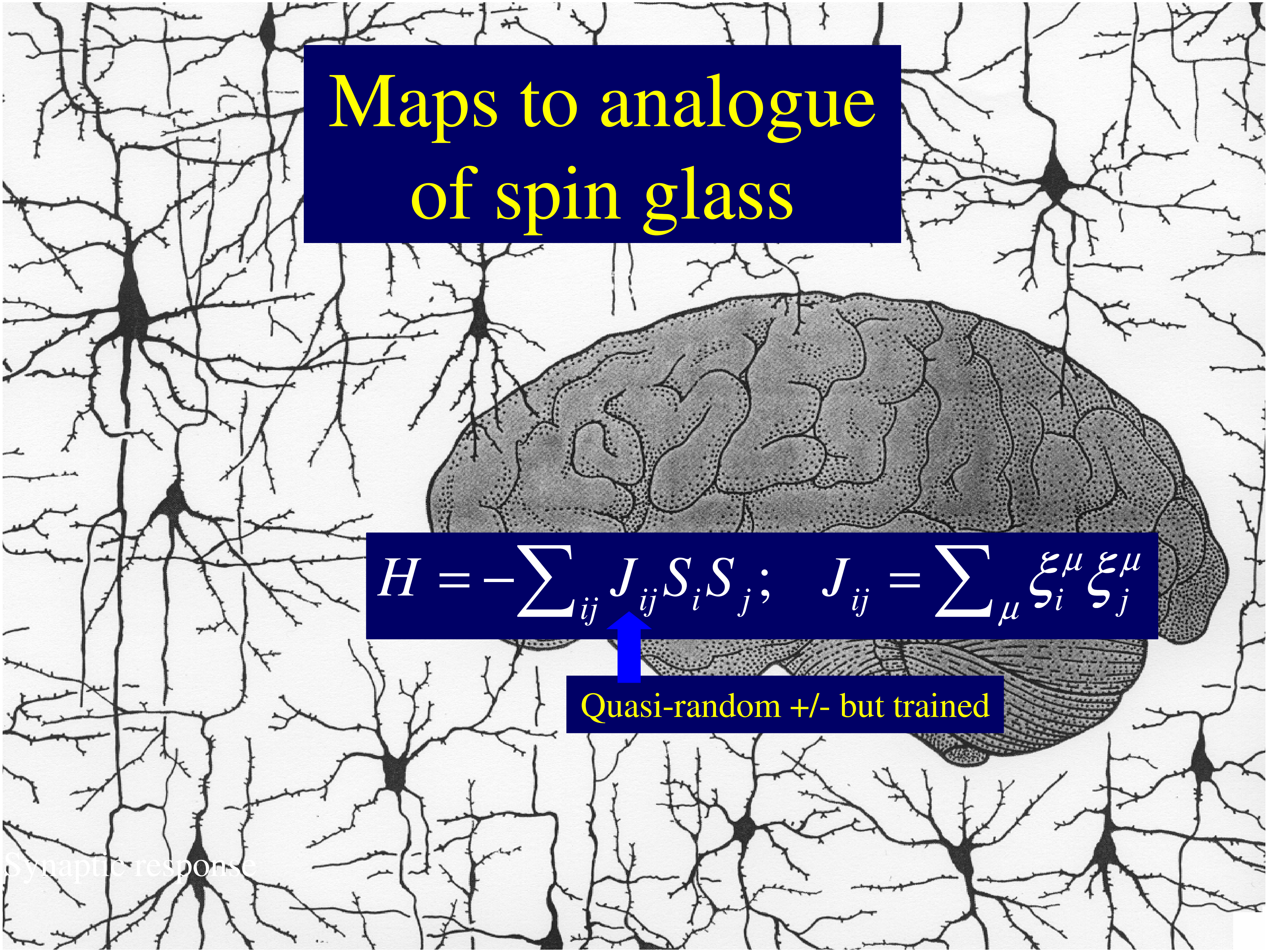


Input potential

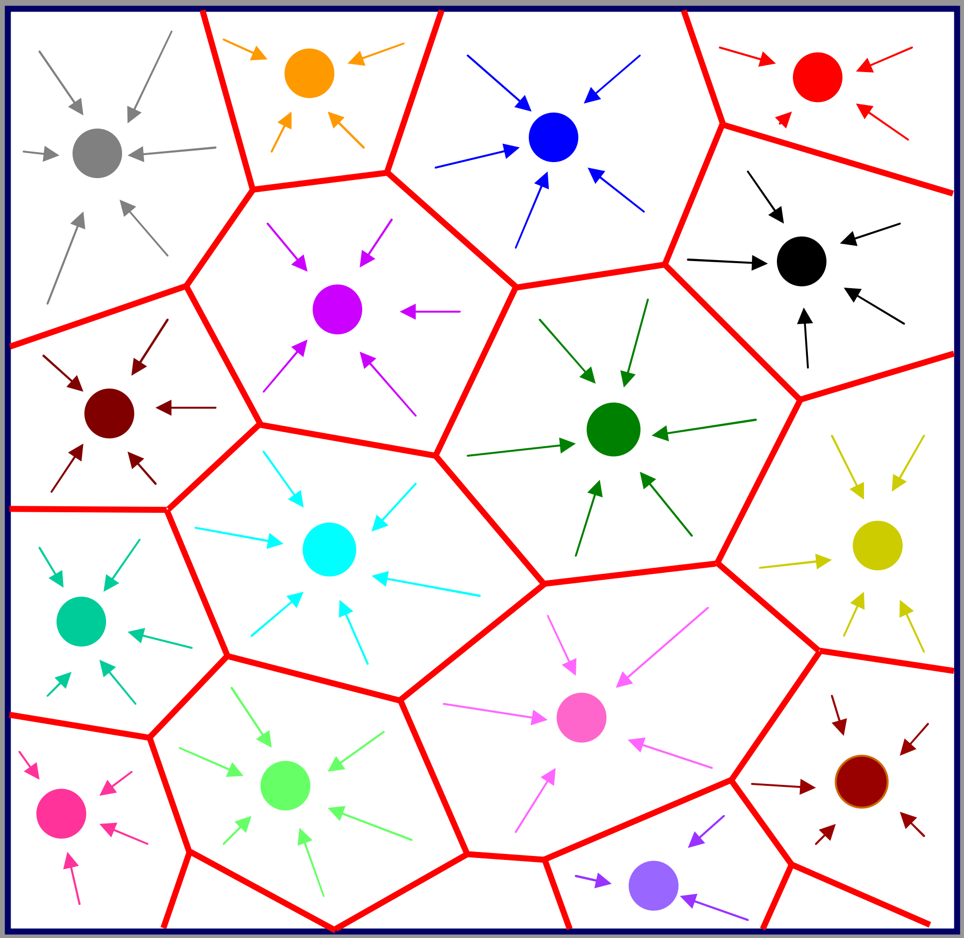
- and so on through the network

Maps to analogue of spin glass

$$H = -\sum_{ij} J_{ij} S_i S_j ; \quad J_{ij} = \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$$



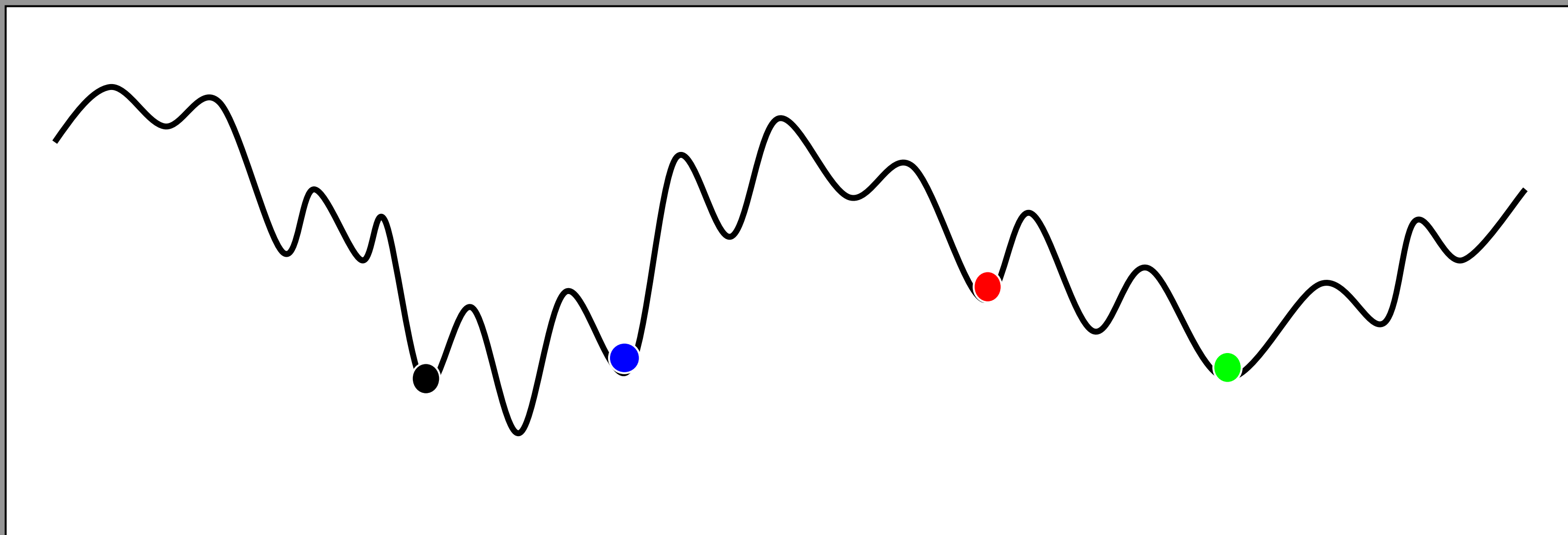
Attractors: tuned metastable states



Phase space

- Associative memory
  - 'attractors'
  - ~ memorized patterns
  - 'basins of attraction' determined by $\{J_{ij}\}$
- Many memories
  - ~ many attractors
  - require frustration

Rugged landscape analogy



Valleys ~ attractors

$\{s_i\}$

Sculpture ~ learning

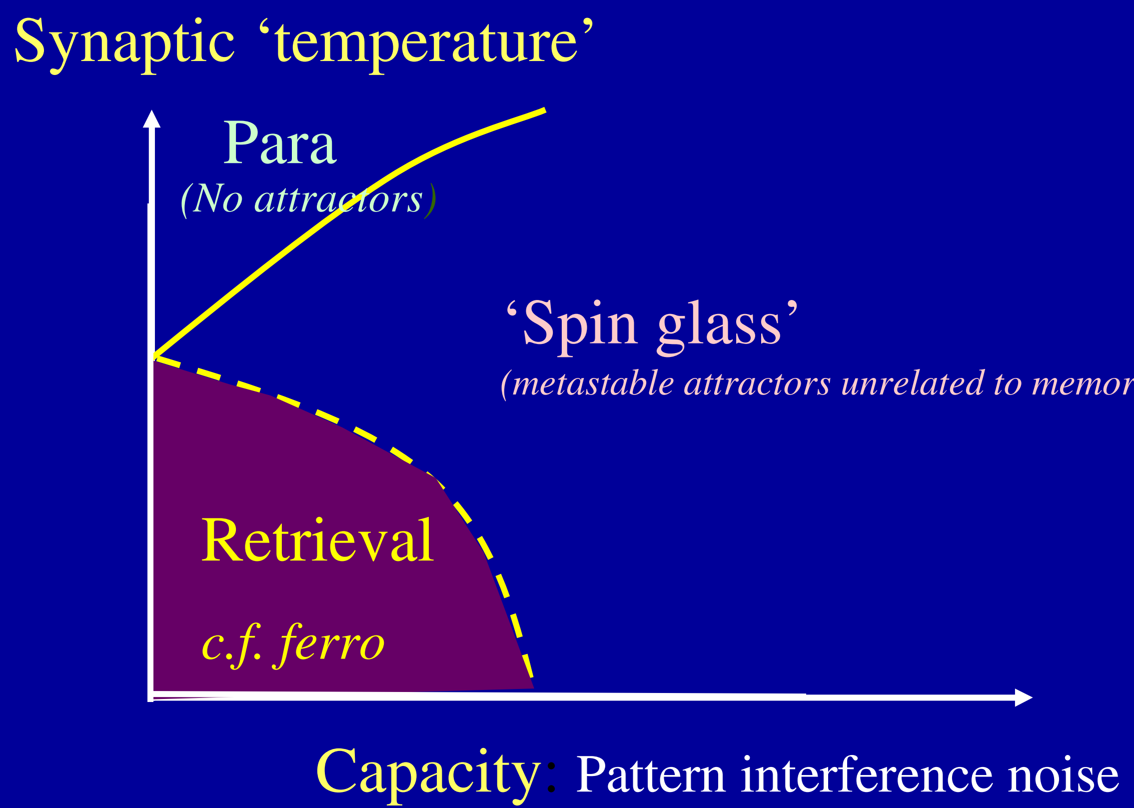
$\{J_{ij}\}$

Different timescales

fast retrieval

slow learning

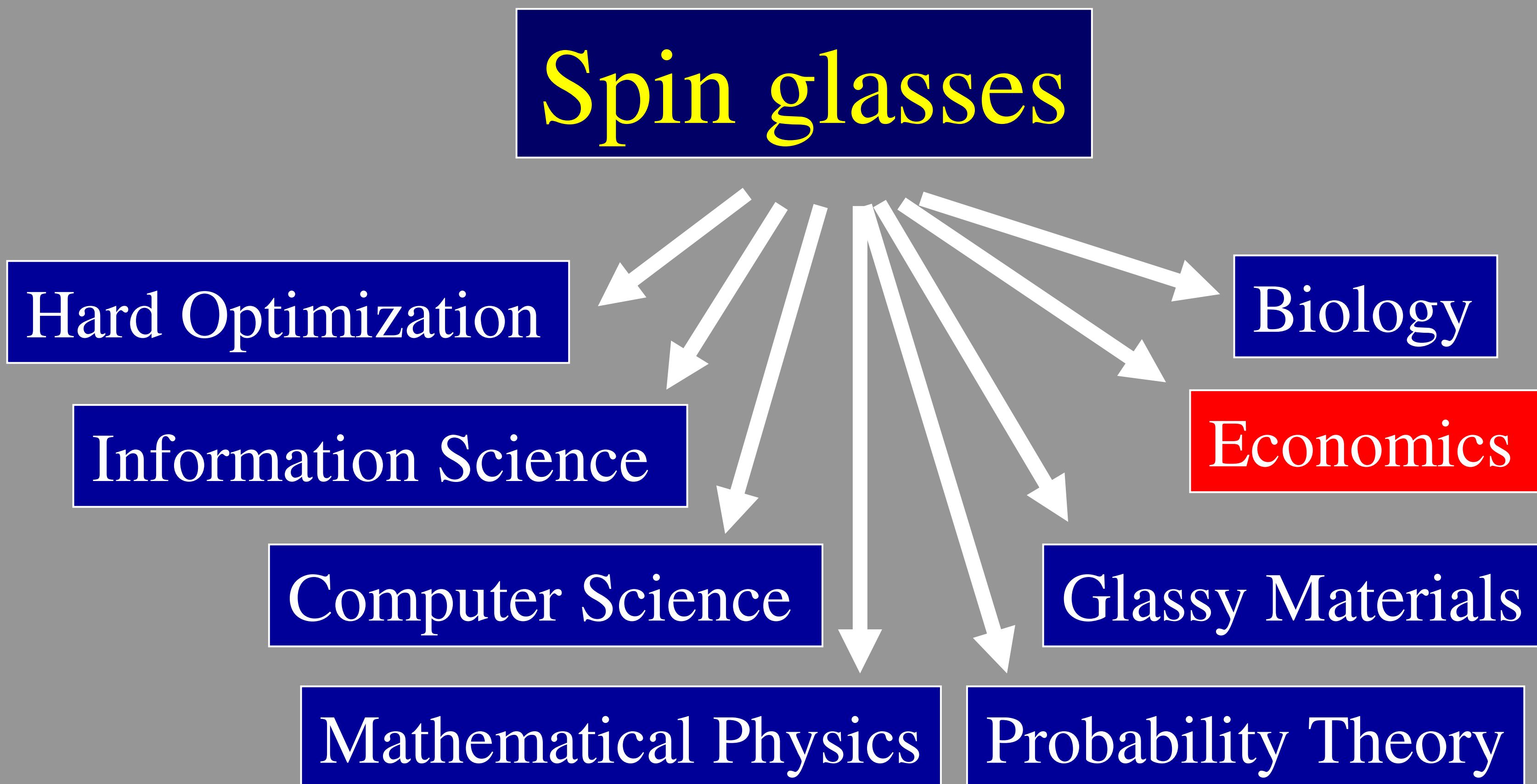
Phase diagram: Hopfield model



Extensions

- Artificial neural networks
  - We design
    - Non-biological elements
    - Train by experience
- Other biological evolution
  - self-train/select
    - maybe without knowing what is “good”
    - e.g. evolution of proteins from heteropolymeric soup
    - Autocatalytic sets

Examples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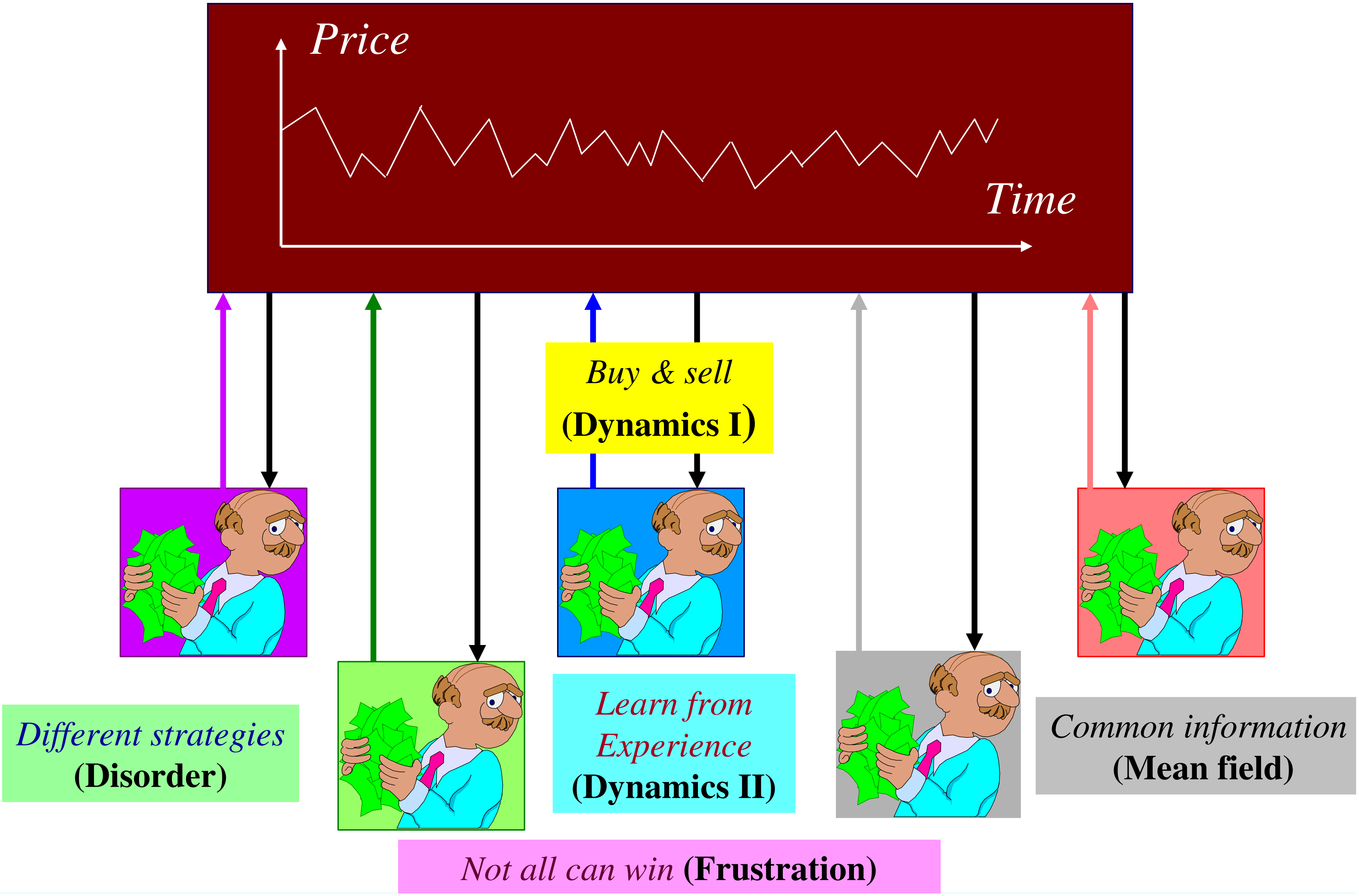
Stockmarket



Minimalist model

Minority game

N agents 2 choices

Aim to be in minority

Individual strategies → Collective consequence

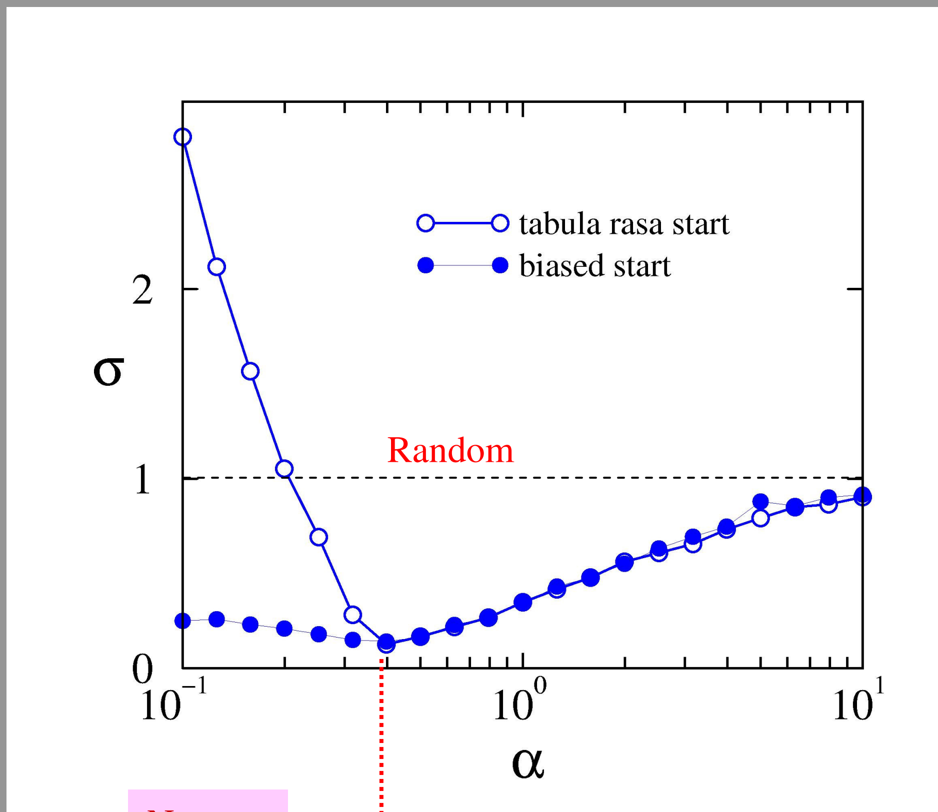
- act on common information (e.g. minority choice for last m steps)
- preferences modified by experience (keep point-score)

↓

Correlated behaviour & phase transition

Phase transition & ergodicity-breaking

Random strategies, random histories



Non-ergodic

Ergodic

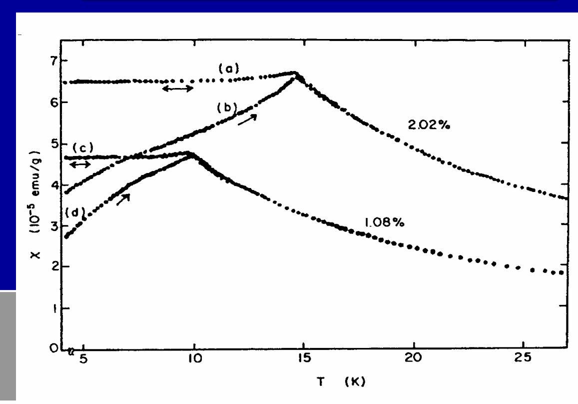
Phase transition: α_c

minimum in volatility

$\alpha < \alpha_c$ *non-ergodic*

$\alpha > \alpha_c$ *ergodic*

c.f. spin glass susc.



Coarse-grained time-average

Effective interaction between agents

$$H = \sum_{ij} J_{ij} s_i s_j + \sum_i h_i s_i$$

Quasi-random *J* and *h* related to agent strategies

c.f. spin glass or neural network

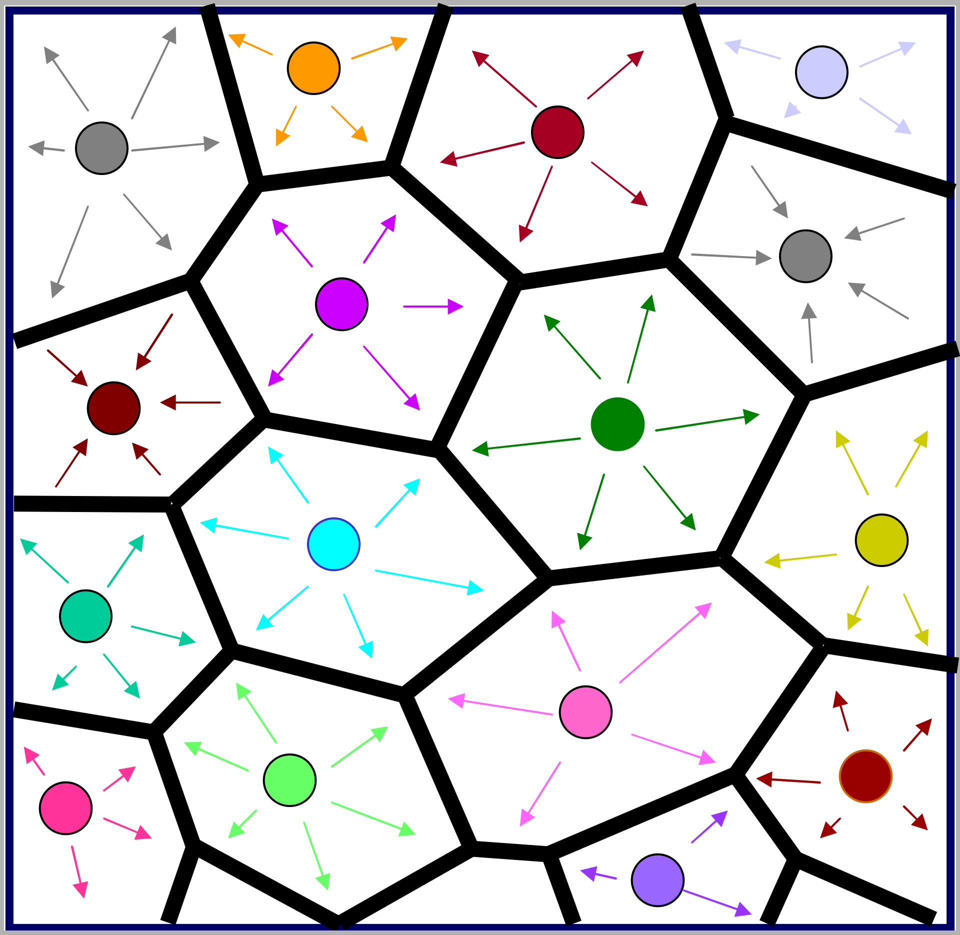
**

Strategy point-score dynamics for agents with 2 strategies

$$p_i(t+1) = p_i(t) - \partial H / \partial s_i \Big|_{\{s_i = \mathrm{sgn}\, p_i(t)\}}$$

Difference from Hopfield neural network

Minority game



$$H = +\sum_{ij} J_{ij} s_i s_j$$

$$J_{ij} = \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}$$

Many repellors

Macrodynamics

Generating functional

Map to macroscopic variables (multi-time)

⬇

Effective ensemble of single agents with

ensemble-self-consistent memory and coloured noise

$$p(t+1) = p(t) - \alpha \sum_{t' \le t} (\mathbf{1}+\mathbf{G})^{-1}{}_{tt'} \,\mathrm{sgn}\, p(t') + \sqrt{\alpha}\,\eta(t)$$

"Representative agent ensemble"

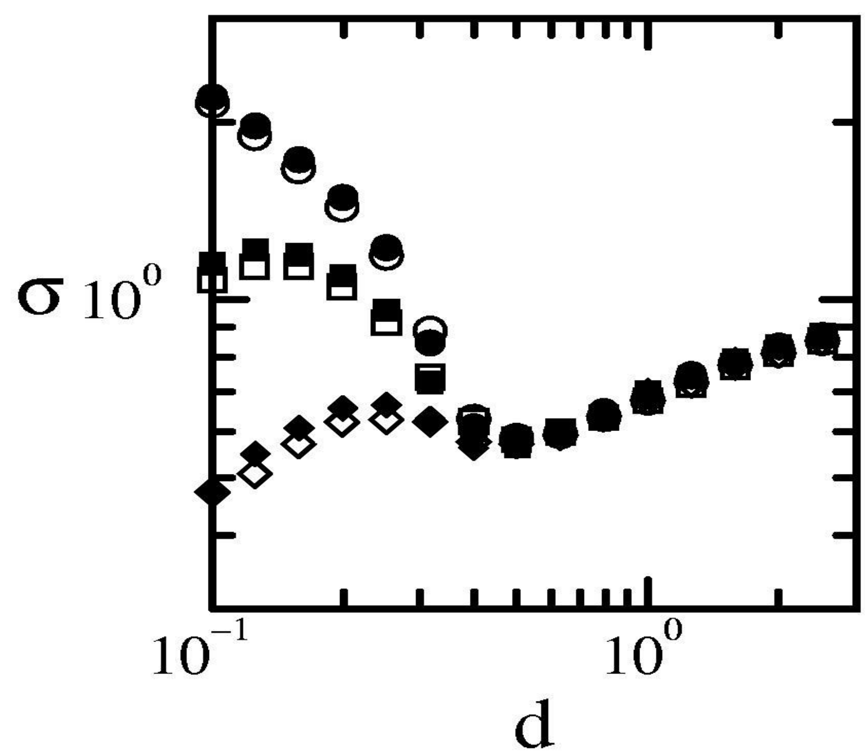
Simulations & iterated theory

Initial bias

$p_i(0)=0$

$p_i(0)=0.5$

$p_i(0)=1$

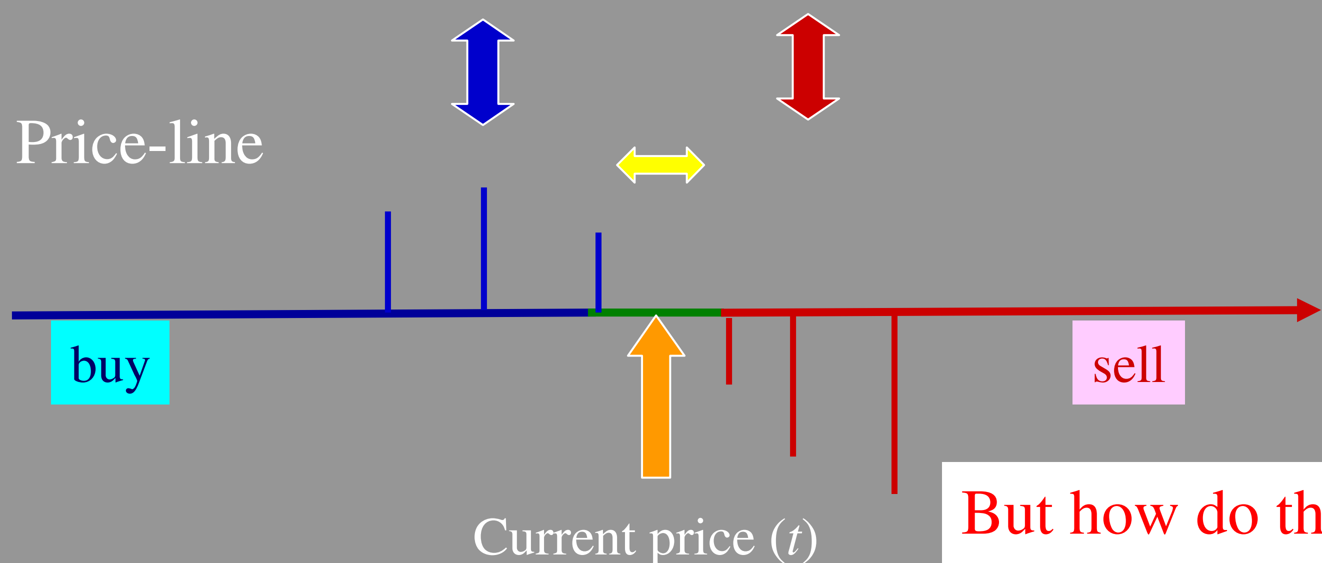


Open = simulations Solid = numerical iteration of analytic effective agent equations

More realistic extension of minority game?

Limit-order book

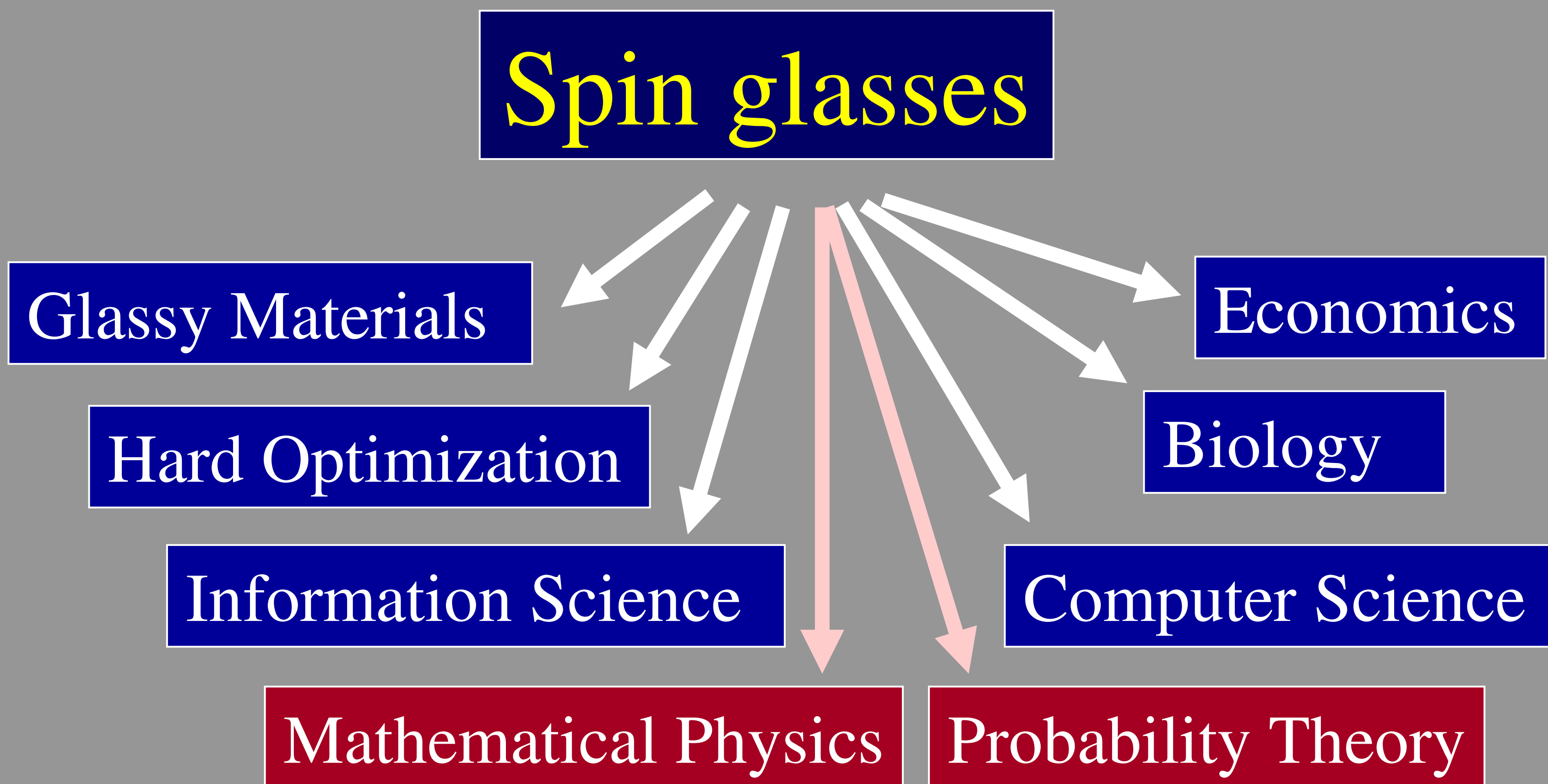
Agents place or remove orders: buy, sell, market. May be executed.
Speculators gain on price changes. Manufacturers must absorb → liquidity.



c.f. Evaporation-deposition-annihilation

But how do they choose what to do?
Evolution of strategies?
Driven by individual attitudes, co-operative actions, learning?

Examples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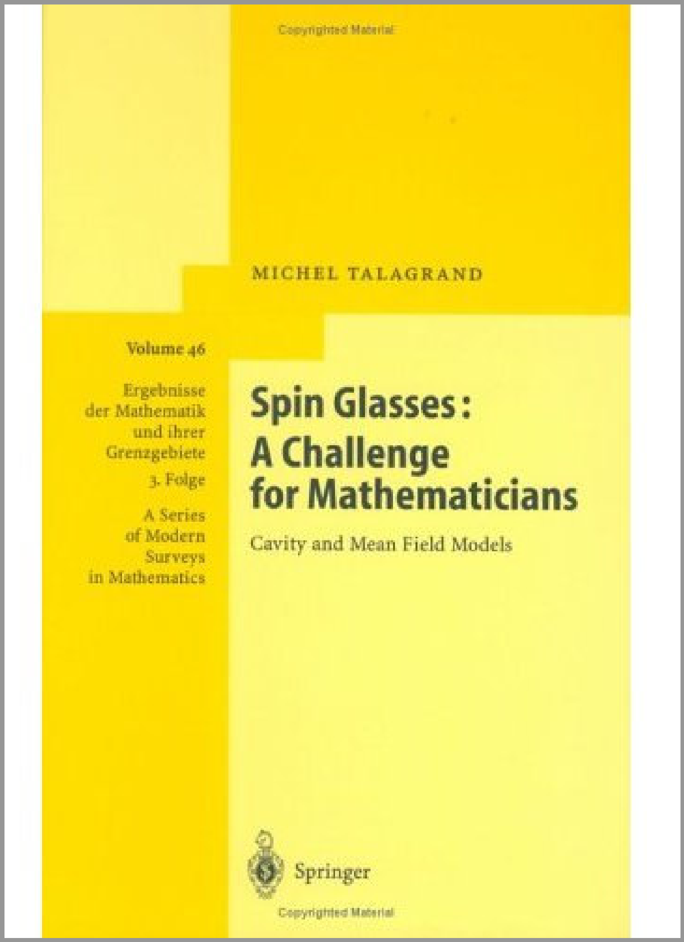
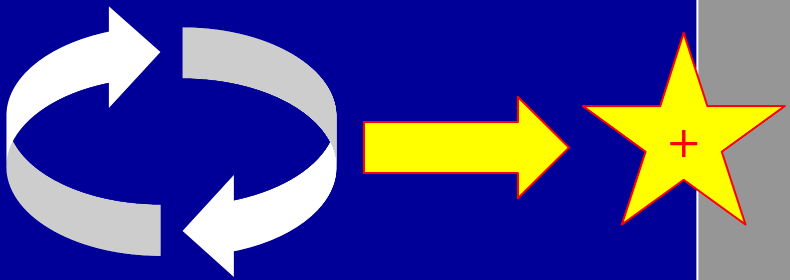
Mathematics & probability



Conclusion I

Symbiosis of techniques

- Theoretical physics interplay
  - Minimalist modelling
  - Sophisticated mathematical analysis
  - Computer simulation
    - Both to check with more complicated real world
    - And to do experiments for which no real analogue
  - Conceptualization
- Real experiment



Useful interdisciplinary transfer

through physics

Not only of

materials and experimental methods

but also of

concepts & mathematical techniques

for

Understanding, quantification & application

And there are many more applications still to consider

Caveats

- I have only given brief indications
  - Needs much fleshing
  - but I hope illustrative of possibilities
- Concentrated on macroscopic properties
  - Not individuals
- And on typical/average behaviour, not fluctuations
  - *e.g.* Not a guide for stockmarket speculation
- But one could do more
  - And there is much more to do

Collaborators

Teachers, colleagues, students, postdocs, friends

Phil Anderson
Sam Edwards
Walter Kohhn

Tomaso Aste
Jay Banavar
Ludovic Berthier
Stefan Boettcher
Arnaud Buhot
Andrea Cavagna
Premla Chandra
Tuck Choy
Ton Coolen
Dinah Cragg
Lexie Davison
Andrea De Martino
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Tobias Galla
Juan Pedro Garrahan
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Irene Giardina
Peter Gillin
Paul Goldbart
Lev Ioffe
Robert Jack
Alexandre Lefevre

Turab Lookman
Peter Kahn
Scott Kirkpatrick
Helmut Katzgraber
Stephen Laughton
Francesco Mancini
Marc Mezard
Esteban Moro
Peter Mottishaw
Normand Mousseau
Hidetoshi Nishimori
Fernando Nobre
Dominic O'Kane

Reinhold Oppermann
Giorgio Parisi
Richard Penney
Albrecht Rau
Avadh Saxena
Manuel Schmidt
Hans-Juergen Sommers
Nicolas Sourlas
Byron Southern
Mike Thorpe
Tim Watkin
Andreas Wendemuth
Werner Wiethege
Stephen Whitelam
Peter Wolynes
Michael Wong

For aficionados

Theoretical methodology

- Statics/thermodynamics:
  - Partition function $$Z = Tr\{\exp[-\beta H]\}$$
- Dynamics:
  - Generating functional

$$Z = \int D\mathbf{S}(t)\delta(\text{microscopic eqn. of motion})$$

* Transform to macrovariables: average over disorder

  Multi-replica/ multi-time correlation & response fns

* Infinite-range

  extremal dominance ~ solubility + subtlety)